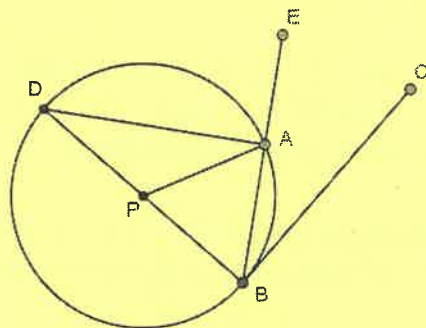


2018 John O'Bryan Mathematical Competition
Junior-Senior Individual Test

Directions: Please answer all questions on the answer sheet provided. All answers must be written legibly and in simplest form. Exact answers are to be given unless otherwise specified in the question. No units of measurement are required. Each problem has the same point-value.

1. If $\frac{a}{c} - 1 = 6\left(\frac{c}{a}\right)$, then $a = kc$. Find the product of all value(s) of k .
2. If $m = \frac{pqr}{p-q}$, solve the equation for q . Assume no denominator is equal to zero. Give your answer as a single fraction.
3. Find the ordered pair of positive prime integers (a, b) with $a \leq b$, for which $a^b \cdot b^a = 6272$.
4. Let $i = \sqrt{-1}$. Find the ordered pair of integers (a, b) with $a > b > 0$ such that $(a + bi)(a - bi) = 13$.
5. Given circle P with tangent \overline{BC} , secant \overline{BE} , and diameter \overline{DB} . If the measure of $\angle ABC = 35^\circ$, find the number of degrees in the measure of the minor arc connecting A and D .



6. If $\sin \theta = \frac{7}{25}$ and $\tan \theta > 0$, find the value of $\cos \theta$. Express your answer as a common fraction reduced to lowest terms.
7. Two distinct numbers are chosen without replacement from the set $\{1, 2, 3, \dots, 30\}$. Find the probability that the sum of the two numbers is odd. Express your answer as a common fraction reduced to lowest terms.
8. Given that $|M|$ represents the determinant of matrix M , solve for x when $\begin{vmatrix} 2 & x & 1 \\ 1 & x & 2 \\ 3 & 4 & 0 \end{vmatrix} = 9$.
9. A sequence is defined recursively as follows: $a_1 = 5$, $a_2 = 9$, $a_n = 2a_{n-1} - a_{n-2}$. Find the value of a_{2018} .
10. Let $i = \sqrt{-1}$. If $x = 4 + i$, $y = -3 + 5i$, and $z = 7 + 4i$, find the value of $\frac{xy-yz}{yx}$. Express your answer as a complex number in the form $a + bi$ where a and b are common or improper fractions reduced to lowest terms.

11. Given that

$$\sum_{n=1}^5 n = 1 + 2 + 3 + 4 + 5, \quad \text{find } \sum_{n=2}^7 (5n + 2^n).$$

12. If $2^{x^2+2x} = 7$, then the exact value of $(x + 1)^2$ is $\log_2 k$ where k is an integer. Find the value of k .

13. If $2x^3 - 3x^2 + kx + 2$ is divisible by $(x - 2)$ and $(x + 1)$, find the value of the sum $(k + w)$.

14. The line with equation $y = k$ is a horizontal asymptote of the graph of

$$y = \frac{3x^2 + 2x}{5 - 7x^2}$$

Find the value of k . Give your answer as a common fraction reduced to lowest terms.

15. If $\frac{\sin(2x)}{\sin(x)} = \frac{k}{w}$ for $0 < x < \frac{\pi}{2}$, write $\cos x$ as a single simplified rational expression in terms of k and w .

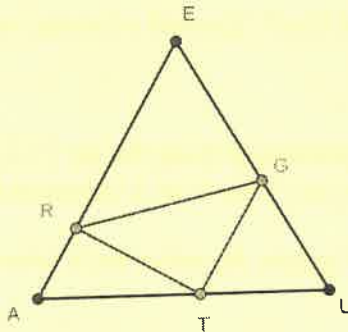
16. Given $\left(x + \frac{1}{x}\right)^2 = 11$ and $x > 0$, find the exact value of $x^3 + \frac{1}{x^3}$. Give your answer in the form $a\sqrt{b}$ where both a and b are integers and b is as small as possible.

17. Find the largest value of x satisfying

$$\binom{2017}{1000} + \binom{2017}{1001} = \binom{2018}{x}$$

Note that $\binom{n}{r} = nCr = C(n, r)$ is the combinatorial combination.

18. $\triangle AEU$ is equilateral, $\triangle RTG$ has a right angle at T , $RA = 1$, $AT = 4$, and $UT = 2$. Find the length of UG . Give your answer as a common or improper fraction reduced to lowest terms.



19. The term $4082400x^2y^5$ appears in the expansion of $(kx + wy)^p$, where k , w , and p are integers. Find the sum $k + w + p$.

20. Circles of radii 4, 5, and 6 are mutually tangent externally. Find the area of the triangle formed by connecting the centers of these three circles. Give your answer in the form $a\sqrt{b}$ where both a and b are integers and b is as small as possible.

Name: _____ **ANSWERS** _____

Team Code: _____

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1. $\frac{-6}{\quad}$

2. $\frac{mp}{m + pr}$

3. $(2,7)$ Must be this ordered pair.

4. $(3,2)$ Must be this ordered pair.

5. 110 (degrees optional)

6. $\frac{24}{25}$ Must be this reduced fraction.

7. $\frac{15}{29}$ Must be this reduced fraction.

8. 7

9. 8073

10. $-\frac{15}{17} - \frac{9}{17}i$ Must be exactly this answer.

11. 387

12. 14

13. -1

14. $-\frac{3}{7}$ Must be this reduced fraction.

15. $\frac{k}{2w}$

16. $8\sqrt{11}$ Must be exactly this answer.

17. 1017

18. $\frac{14}{5}$ Must be this reduced fraction.

19. 18

20. $30\sqrt{2}$ Must be exactly this answer.